

Gravitino production during preheating*

Antonio L. Maroto[†]
Astronomy Centre, University of Sussex,
Falmer, Brighton, BN1 9QJ, U.K.

February 1, 2008

Abstract

We study the production of gravitinos during the preheating era after inflation by means of the non-perturbative Bogolyubov technique. Considering only the helicity $\pm 3/2$ states, the problem is reduced to the simpler Dirac fermions case. We calculate the production in a particular supergravity model in an expanding universe and obtain the spectrum and number density. Finally we compare the results with the nucleosynthesis bounds and extract some consequences.

1 Introduction

The presence of weakly interacting massive particles can have important consequences in cosmology [1]. In particular, when such particles are light, with life times longer than the universe age, they can act as dark matter or even dominate the present energy density and overclose the universe. On the other hand, if they are unstable and decay during the nucleosynthesis era, they could destroy the nuclei created in this period and spoil the successful predictions of the standard big bang model.

*Talk given in the International Workshop on Particles in Astrophysics and Cosmology, Valencia (Spain), May 3-8, 1999. To appear in the proceedings.

[†]Present address: Dept. Física Teórica, Universidad Complutense de Madrid, 28040, Madrid, Spain. E-mail: maroto@eucmax.sim.ucm.es

One of such particles is the gravitino, i.e, the spin $3/2$ superpartner of the graviton in supergravity theories, whose couplings to the rest of particles are suppressed by the Planck mass scale. Typically its decay rate is given by $\Gamma_{3/2} \simeq m_{3/2}^3/M_P^2$ which implies that gravitinos lighter than $m_{3/2} < 100\text{MeV}$ will live longer than the universe age. Due to their weak couplings, gravitinos freeze-out very early when they are still relativistic and therefore their primordial abundance can be estimated as $n_{3/2}/s \simeq 10^{-3}$ [1]. This allows us to obtain the well-known bound on the mass of stable gravitinos, i.e, in order not to overclose the universe we need $m_{3/2} < 1\text{keV}$ [2]. However if they are unstable, that primordial abundance would give rise to an enormous amount of entropy that will conflict with the standard cosmology. A possible way out of this *gravitino problem* is the existence of a period of inflation that dilutes any primordial density [3]. Unfortunately the problem can be recreated if after inflation, gravitinos are produced by some mechanism. In fact, this could be the case if during the period of inflaton oscillations at the end of inflation, the reheating temperature was sufficiently high. A successful nucleosynthesis period requires (we give some conservative bounds [4]): $n_{3/2}/s < 10^{-15}$ for a gravitino mass $m_{3/2} \simeq 100\text{GeV}$, $n_{3/2}/s < 10^{-14}$ for $m_{3/2} \simeq 1\text{TeV}$ and $n_{3/2}/s < 10^{-13}$ for $m_{3/2} \simeq 10\text{TeV}$. The production of gravitinos during reheating depends on the temperature T_R as [5]: $n_{3/2}/s \simeq 10^{-14}T_R/(10^9\text{GeV})$, which implies that for a typical mass $m_{3/2} \simeq 1\text{TeV}$, the reheating temperature has to be $T_R < 10^9\text{GeV}$. Another constraint appears in supergravity models, where the gravitino mass is determined by the scale of supersymmetry breaking. In order to solve the hierarchy problem, it is then suggested that $m_{3/2} < 1\text{TeV}$ [6].

As we have just mentioned, during the reheating period gravitinos can be created by the perturbative decay of other particles produced from the inflaton oscillations. However, in recent years, the standard picture of reheating has changed dramatically as a consequence of some works in which it was realized that during the first inflaton oscillations, reheating can not be studied by the standard perturbative techniques [7, 8]. This so called *preheating* period can give rise to an explosive production of bosons due to the phenomenon of parametric resonance. In this period, the energy of the coherent oscillations of the inflaton field is very efficiently converted into particles. In the case of fermions, the limit imposed by Pauli exclusion principle avoids the explosive production, but however the results deviate from the perturbative expectations [9, 10, 11]. This fact will be of the utmost importance

in the case that gravitinos are directly coupled to the inflaton, since the new preheating period can give rise to an excess of production.

In this work we present our results on the production of spin 3/2 particles from the non-perturbative Bogolyubov technique [12] and apply the method to the preheating period in a particular supergravity model.

2 Quantization of spin 3/2 fields in external backgrounds

Preheating is based on the phenomenon of creation of particles from classical backgrounds (see [13] and references therein). The spin 3/2 case is however special due to the consistency problems that avoid the quantization of such fields in the presence of scalar, electromagnetic or gravitational backgrounds [14]. The only theory in which these problems seem to be absent is supergravity, provided the background fields satisfy the corresponding equations of motion [15]. But, the complicated form of the Rarita-Schwinger (RS) equation makes it very difficult to extract explicit results even in simple cases. Let us start by considering the massive RS equation in a curved space-time. We will include the coupling to a scalar field χ by modifying the mass term, (we will follow the notation in [16]):

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho\psi_\sigma + \frac{1}{2}(m_{3/2} - \chi)[\gamma^\mu, \gamma^\nu]\psi_\nu = 0. \quad (1)$$

As usual in supergravity models we will consider Majorana spinors satisfying $\psi_\mu = C\bar{\psi}_\mu^T$ with $C = i\gamma^2\gamma^0$ the charge conjugation matrix. We have introduced the curvature of space-time by *minimal* coupling as done in supergravity, i.e, $D_\rho\psi_\sigma = (\partial_\rho + \frac{i}{2}\Omega_\rho^{ab}\Sigma_{ab})\psi_\sigma$ with Ω_ρ^{ab} the spin-connection coefficients and $\Sigma_{ab} = \frac{i}{4}[\gamma_a, \gamma_b]$. The $\epsilon^{\mu\nu\rho\sigma}$ removes the Christoffel symbols contribution in the covariant derivative. Following the analogy with the creation of gravitons from Einstein equations, we will consider only the linearized equation in $1/M$ ($M_P^2 = 8\pi M^2$) for supergravity [6], i.e, we will consider only the symmetric part of the spin-connection, ignoring the torsion contribution that is $\mathcal{O}(M^{-2})$. In flat space-time and when $\chi = 0$, the general solution of these equations can be expanded in four helicity $l = s/2 + m$ modes

($l = \pm 3/2, \pm 1/2$):

$$\psi_\mu^{pl}(x) = e^{-ipx} \sum_{s,m} J_{sm} u(\vec{p}, s) \epsilon_\mu(\vec{p}, m) \quad (2)$$

with J_{sm} the Clebsch-Gordan coefficients whose values are: $J_{-1-1} = J_{11} = 1$, $J_{-11} = J_{1-1} = 1/\sqrt{3}$ and $J_{-10} = J_{10} = \sqrt{2/3}$. $u(\vec{p}, s)$ are spinors with definite helicity $s = \pm 1$ and $\epsilon_\mu(\vec{p}, m)$ with $m = \pm 1, 0$ are the three spin 1 polarization vectors.

In order to study the preheating era, we need to consider the presence of the inflaton field and the expanding universe. With that purpose we take a scalar field only depending on time $\chi(t)$ and a spatially flat Friedmann-Robertson-Walker metric.

Now, the expression in (2) is not a solution of the equations of motion. However, we can look for general homogeneous solutions of the RS equation in the form:

$$\psi_\mu^{pl}(x) = e^{i\vec{p}\cdot\vec{x}} f^{pl}(t) \sum_{s,m} J_{sm} u(\vec{p}, s) \epsilon_\mu(\vec{p}, m) \quad (3)$$

These fields satisfy the condition $\gamma^\mu \psi_\mu = 0$ and if we constraint ourselves to the helicity $l = \pm 3/2$ states, then the RS equation reduces to a Dirac form [12]:

$$(i \not{D} - m_{3/2} + \chi) \psi_\mu = 0 \quad (4)$$

and the condition $D^\mu \psi_\mu = 0$ also holds. As far as these modes satisfy a Dirac-like equation, it suggests that all the difficulties in the gravitino quantization procedure would reduce to the helicity $\pm 1/2$ modes in this case. In fact the above ansatz (3) is not a solution for the helicity $\pm 1/2$ modes even for homogeneous backgrounds.

Thus we see that the production of helicity $\pm 3/2$ gravitinos during preheating in an expanding universe can be studied in the same way as the production of Dirac fermions. With this purpose we have to reduce equation (4) to a second order equation. Let us first write the equation in conformal time defined as $dt = a(\eta)d\eta$:

$$\left(ia^{-1} \gamma^\mu \partial_\mu - m_{3/2} + \chi + i \frac{3}{2} \frac{\dot{a}}{a^2} \gamma^0 \right) \psi_\mu = 0 \quad (5)$$

where $\dot{a} = da/d\eta$. We will take the following ansatz on the helicity $l = \pm 3/2$ solutions:

$$\psi_\mu^{pl}(x) = a^{-3/2}(\eta) e^{i\vec{p}\cdot\vec{x}} U_\mu^{\vec{p}l}(\eta) \quad (6)$$

with

$$\begin{aligned} U_\mu^{\vec{p}l}(\eta) = & \frac{1}{\sqrt{\omega + m_{3/2}^0}} \left(i\gamma^0 \partial_0 - \vec{p} \cdot \vec{\gamma} \right. \\ & \left. + a(\eta)(m_{3/2} - \chi(\eta)) f_{pl}(\eta) u(\vec{p}, s) \epsilon_\mu(\vec{p}, m) \right) \end{aligned} \quad (7)$$

and the normalization $U_\mu^{\vec{p}l\dagger}(0) U_\mu^{\vec{p}l}(0) = 2\omega$ where $m_{3/2}^0 = a(0)m_{3/2}$. It is possible to check that this ansatz automatically satisfies $\gamma^\mu \psi_\mu = 0$ and $D^\mu \psi_\mu = 0$. An appropriate form for the spinor $u(\vec{p}, s)$ and polarization vectors $\epsilon_\mu(\vec{p}, m)$ can be obtained if we choose the Dirac representation for the gamma matrices and we take (without loss of generality) the z -axis along the \vec{p} direction. In this case $u(\vec{p}, 1)^T = (1, 0, 0, 0)$, $u(\vec{p}, -1)^T = (0, 1, 0, 0)$, $\epsilon_\mu(\vec{p}, 1) = \frac{1}{\sqrt{2}}(0, 1, i, 0)$ and $\epsilon_\mu(\vec{p}, -1) = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$. With this choice $u(\vec{p}, \pm 1)$ are eigenstates of γ^0 with eigenvalues ± 1 . Then equation (5) reduces to the well-known form:

$$\begin{aligned} \left(\frac{d^2}{d\eta^2} + p^2 - i \frac{d}{d\eta} \left(a(\eta)(m_{3/2} - \chi(\eta)) \right) \right. \\ \left. + a^2(\eta)(m_{3/2} - \chi(\eta))^2 \right) f_{pl}(\eta) = 0 \end{aligned} \quad (8)$$

In order to quantize the modes we will expand an arbitrary solution with helicity $l = \pm 3/2$ as:

$$\psi_\mu^l(x) = \int \frac{d^3p}{(2\pi)^3 2\omega} a^{-3/2}(\eta) \left(e^{i\vec{p}\cdot\vec{x}} U_\mu^{\vec{p}l}(\eta) a_{\vec{p}l} + e^{-i\vec{p}\cdot\vec{x}} U_\mu^{\vec{p}lC}(\eta) a_{\vec{p}l}^\dagger \right) \quad (9)$$

where the creation and annihilation operators satisfy the anticommutation relations $\{a_{\vec{p}l}, a_{\vec{p}'l'}^\dagger\} = (2\pi)^3 2\omega \delta_{ll'} \delta(\vec{p} - \vec{p}')$.

3 A supergravity inflation example

We will consider a specific supergravity inflationary model (see [17]), in which the inflaton field is taken as the scalar component of a chiral superfield,

and its potential is derived from the superpotential $I = (\Delta^2/M)(\phi - M)^2$. This is the simplest choice that satisfies the conditions that supersymmetry remains unbroken in the minimum of the potential and that the present cosmological constant is zero. CMB anisotropy fixes the inflationary scale around $\lambda \equiv \Delta/M \simeq 10^{-4}$. For the sake of simplicity, we will consider the case in which the gravitino mass is much smaller than the effective mass of the inflaton in this model, $m_{3/2} \ll m_\phi \simeq 10^{-8}M$ and since the production will take place during a few inflaton oscillations, we will neglect the mass term in the equations. The scalar field potential is shown to be stable in the imaginary direction and therefore we will take for simplicity a real inflaton field. Along the real direction the potential can be written as:

$$V(\phi) = \lambda^4 e^{\phi^2} \left((2(\phi - 1) + \phi(\phi - 1)^2)^2 - 3(\phi - 1)^4 \right) \quad (10)$$

where we are working in units $M = 1$. This potential has a minimum in $\phi = 1$. The coupling of the inflaton field to gravitinos is given by the following term in the supergravity lagrangian [6]:

$$\mathcal{L}_{int} = -\frac{1}{4} e^{G/2} \bar{\psi}_\mu [\gamma^\mu, \gamma^\nu] \psi_\nu \quad (11)$$

$$e^{G/2} = \lambda^2 e^{\phi^2/2} (\phi - 1)^2 \quad (12)$$

where the Kähler potential has been chosen in such a way that the kinetic terms for the scalar fields are canonical $G(\Phi, \Phi^\dagger) = \Phi^\dagger \Phi + \log |I|^2$. The inflaton and Friedmann equations can be written in conformal time as:

$$\ddot{\phi} + 2\frac{\dot{b}}{b}\dot{\phi} + \frac{b^2}{\lambda^4} V_{,\phi} = 0 \quad (13)$$

$$\frac{\dot{b}^2}{b^2} = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + \frac{b^2}{\lambda^4} V \right) \quad (14)$$

where the derivatives are with respect to the new time coordinate $\tilde{\eta} = a_0 \lambda^2 \eta$ and the new scale factor is defined as $b(\tilde{\eta}) = a(\tilde{\eta})/a_0$ with $a_0 = a(0)$. The solution of this equation shows that after the inflationary phase, the scalar field starts oscillating around the minimum of the potential with damped amplitude. Substituting in (8) and taking $\chi = e^{G/2}$ for this particular case, we obtain:

$$\left(\frac{d^2}{d\tilde{\eta}^2} + \kappa^2 + \frac{i}{\lambda^2} \frac{d}{d\tilde{\eta}} (b e^{G/2}) + \frac{b^2}{\lambda^4} e^G \right) f_{\kappa l}(\tilde{\eta}) = 0 \quad (15)$$

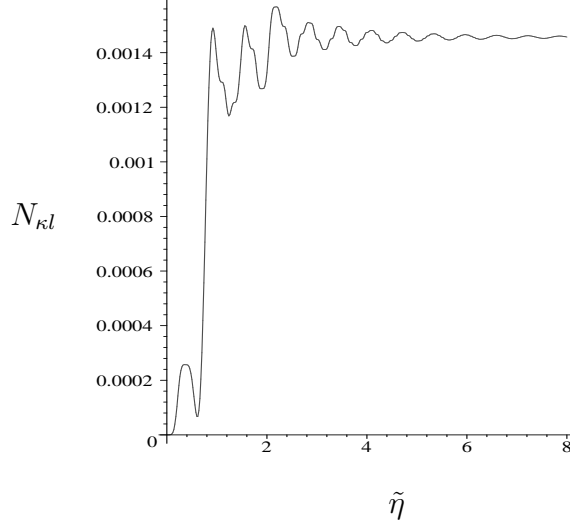


Figure 1.—Number of helicity $l = \pm 3/2$ gravitinos ($N_{\kappa l}$) as a function of time $\tilde{\eta}$ for $\kappa = 5$

with $\kappa = p/(a_0\lambda^2)$. From this expression we see that when the scalar interaction is switched off, even in the expanding background, there is no particle production. This is not surprising since in that case, the equations of motion (in the massless limit) are conformally invariant. Following [9, 13] we can calculate the occupation number:

$$N_{\kappa l}(\tilde{T}) = \frac{1}{4\kappa} \left(2\kappa + i[f_{\kappa l}^*(\tilde{T})\dot{f}_{\kappa l}(\tilde{T}) - f_{\kappa l}(\tilde{T})\dot{f}_{\kappa l}^*(\tilde{T})] - \frac{2}{\lambda^2} b e^{G(\tilde{T})/2} |f_{\kappa l}(\tilde{T})|^2 \right) \quad (16)$$

In order for the particle number to be well defined, we will evaluate it when the interaction is vanishingly small, that is, for large values of \tilde{T} . Here $f_{\kappa l}$ is a solution of equation (15) with initial conditions $f_{\kappa l}(0) = 1$ and $\dot{f}_{\kappa l}(0) = -i\kappa$ which corresponds to a plane wave for $\tilde{\eta} \leq 0$. In order to define the initial vacuum at $\tilde{\eta} = 0$, we have taken the inflaton to be at the minimum of the potential at that moment ($\phi(0) = 1$), which implies that the interaction term vanishes, i.e., $e^{G(\phi=1)/2} = 0$. According to the definition of b we also have $b(0) = 1$. We have chosen $\dot{\phi}(0) = 1.8$ in our numerical computations which corresponds to an initial amplitude of the inflaton oscillations around $0.06M_p$.

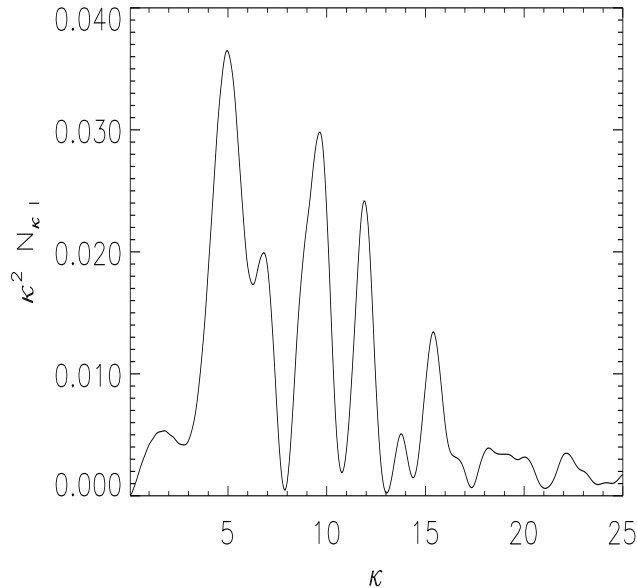


Figure 2.-Number density of helicity $l = \pm 3/2$ gravitinos ($\kappa^2 N_{\kappa l}$) against κ .

In Fig.1, the behaviour of $N_{\kappa l}$ has been plotted as a function of time. We see how the production takes place in a few inflaton oscillations and that asymptotically the number of particles created is a well-defined quantity since the interaction vanishes. The results for the spectra in the expanding background can be found in Fig.2. Notice that we have not considered the backreaction effect of the produced particles on the scalar field evolution. Because of the expansion of the universe, the production is significantly reduced with respect to the flat space case. As expected [7] the resonance structure is affected by the expansion and the Pauli limit is not saturated, but the number of particles that are produced is not negligible.

We expect modifications in the spectrum for different initial conditions, although the total number of particles created is not very sensitive to these changes. From Fig. 2, we can estimate a lower bound to the total number density of gravitinos with both helicities as:

$$n_{pre}(\eta) = \frac{1}{\pi^2 a^3(\eta)} \int_{p_{min}}^{\infty} N_{pl} p^2 dp = \frac{a_0^3 \lambda^6}{\pi^2 a^3} \int_{\kappa_{min}}^{\infty} N_{\kappa l} \kappa^2 d\kappa \quad (17)$$

with $p_{min} = 2\pi H(\eta)$. Since today, $(\eta_0, a(\eta_0) = 1)$ $H \ll Ma_0\lambda^2$, we get: $n_{pre}(\eta_0) \geq a_0^3 10^{28} GeV^3$. To be compared with the number density of a thermal distribution of helicity $\pm 1/2$ gravitinos as estimated in [2] (the helicity $\pm 3/2$ could be even less dense): $n(\eta_0) \simeq 10^{-40} GeV^3$. The comparison depends on the dilution term a_0^3 , i.e. on the scale factor at the end of inflation, and it shows that, for example, for a typical value [1] $a_0 \simeq 10^{-26}$, the vacuum fluctuation production is suppressed with respect to the thermal distribution by a factor $\geq 10^{-10}$. The corresponding cosmological consequences have been studied in [3]. Comparing with the entropy density today we get: $n_{pre}/s \geq 10^{-12}$.

If we compare these results with the nucleosynthesis bounds given in the introduction, we see that the preheating production is compatible with them for a mass of (unstable) gravitino $m_{3/2} > \mathcal{O}(10 TeV)$ for this particular model. This implies that gravitino masses $100 MeV < m_{3/2} < 10 TeV$ would be forbidden. On the other hand, if gravitinos are stable particles, then in order not to overclose the universe we get: $m_{3/2} < 1 TeV$. These bounds should be taken with caution, as the results strongly depend on the model parameters.

In the standard scenario of reheating for this model, the reheating temperature is very low $T_R \simeq 10^5 GeV$ which implies that the perturbative gravitino production, that can take place either from $2 \rightarrow 2$ processes involving gauge bosons and gauginos or from direct inflaton decay [17], is very small $n_{reh}/s \simeq \mathcal{O}(10^{-17})$. Comparing with the preheating results above, we see that the difference between the two mechanisms is apparent in these estimations.

Acknowledgements: I thank A. Mazumdar for useful discussions. This work has been supported by SEUID-Royal Society and CICYT-AEN97-1693 (Spain).

References

- [1] E.W. Kolb and M.S Turner, *The Early Universe* (Addison-Wesley) (1990)
- [2] H. Pagels and J. Primack, *Phys. Rev. Lett.* **48** 223 (1982)
- [3] J. Ellis, A. Linde and D. Nanopoulos, *Phys. Lett.* **118B**, 59 (1982)

- [4] S. Sarkar, *Rep. Prog. Phys.* **59**, 1493 (1996)
- [5] J. Ellis, J.E. Kim and D.V. Nanopoulos, *Phys. Lett.* **145B**, 181 (1984)
- [6] D. Bailin and A. Love, *Supersymmetric Gauge Field Theory and String Theory*, IOP, Bristol, (1994)
- [7] L. Kofman, A. D. Linde and A.A. Starobinsky, *Phys. Rev. Lett.* **73** (1994) 3195; L.Kofman, A.D. Linde and A. A. Starobinsky, *Phys. Rev. D* **56** (1997) 3258
- [8] J.H. Traschen, R.H. Brandenberger, *Phys. Rev.* **D42** (1990), 2491; Y. Shtanov, J. Traschen and R. Brandenberger, *Phys. Rev.* **D51** (1995), 5438
- [9] J. Baacke, K. Heitmann and C. Patzold, *Phys. Rev.* **D58** 125013 (1998)
- [10] P.B. Greene and L. Kofman, *Phys. Lett.* **B448**, 6 (1999)
- [11] A.L. Maroto and A. Mazumdar, *Phys. Rev.* **D59**, 083510 (1999)
- [12] A.L. Maroto and A. Mazumdar, hep-ph/9904206
- [13] A.A. Grib, S.G. Mamayev and V.M. Mostepanenko *Vacuum Quantum Effects in Strong Fields*, Friedmann Laboratory Publishing, St. Petersburg (1994)
- [14] G. Velo and D. Zwanziger, *Phys. Rev* **186** (1969) 1337; C.R. Hagen and L.P.S. Singh, *Phys. Rev. D* **26** (1982) 393
- [15] S. Deser and B. Zumino, *Phys. Rev. Lett.* **38** (1977) 1433; *Phys. Lett.* **62B**, 335 (1976)
- [16] M. Kawasaki and T. Moroi, *Prog. Theor. Phys.* **93**, 879 (1995); T. Moroi, PhD Thesis (1995), hep-ph/9503210
- [17] G.G. Ross and S. Sarkar, *Nucl. Phys.***B461** (1996) 597